Jurnal Inovasi dan Teknologi Pendidikan
JURINOTEP
Vol. 4, No. 2, September, 2025 hal. 627-637
Journal Page is available to https://lppmbinabangsa.id/index.php/jurinotep

Commognitive Review: Analysis of Student Work Results in Solving Contextual Problems on Matrix Inverse Material

Amalia Chamidah[1*], Endrayana Putut Laksminto Emanuel[2]
[1,2]Universitas Wijaya Kusuma Surabaya
amaliachamidah_fbs@uwks.ac.id

Article Info

Article History
Received : 28-08-2025
Revised : 13-09-2025
Accepted : 13-09-2025

Kata Kunci : commognitive, analisis commognitive, hasil kerja mahasiswa, soal kontekstual, invers matriks

Abstrak

Soal kontekstual matematika sering kali dianggap sulit oleh mahasiswa karena menuntut pemahaman konsep dan kemampuan mengaitkan matematika dengan situasi nyata. Penelitian ini bertujuan untuk menganalisis hasil kerja mahasiswa dalam menyelesaikan soal kontekstual mengenai invers matriks. Penelitian ini menggunakan pendekatan kualitatif dengan teknik purposive sampling. Sebanyak 20 mahasiswa berpartisipasi dalam penelitian dan diminta menyelesaikan satu soal kontekstual secara individu dalam waktu 30 menit. Hasil pekerjaan mahasiswa dikelompokkan berdasarkan kebenaran jawaban, kemudian dua subjek dari kelompok jawaban salah dipilih secara purposive untuk dianalisis lebih mendalam menggunakan kerangka kerja commognitive. Data diperkuat melalui wawancara semi terstruktur guna mengklarifikasi proses berpikir mahasiswa dalam menyelesaikan soal. Hasil penelitian menunjukkan bahwa kurangnya pemahaman terhadap pengetahuan prasyarat yang diperoleh di jenjang sekolah menengah menyebabkan terjadinya kesalahan dalam langkah penyelesaian soal (RR). Selain itu, penggunaan aturan atau prosedur yang tidak sesuai juga memunculkan kesalahan konseptual (NM). Oleh karena itu, pengulangan dan penguatan materi prasyarat sangat diperlukan untuk meningkatkan pemahaman mahasiswa dan meminimalkan kesalahan dalam menyelesaikan soal kontekstual.

Keywords: commognitive, commognitive analysis, student work results, contextual problems, inverse matrix

Contextual mathematics problems are often considered difficult by students because they require conceptual understanding and the ability to relate mathematics to real-world situations. This study aims to analyze students' work in solving contextual problems concerning matrix inverses. This study used a qualitative approach with a purposive sampling technique. A total of 20 students participated in the study and were asked to solve one contextual problem individually within 30 minutes. The students' work was grouped based on the correctness of their answers, then two subjects from the incorrect answer group were purposively selected for further analysis using a comcognitive framework. Data were strengthened through semi-structured interviews to clarify students' thinking processes in solving the problems. The results showed that a lack of understanding of prerequisite knowledge acquired in secondary school led to errors in problem-solving steps (RR). In addition, the use of inappropriate rules or procedures also gave rise to conceptual errors (NM). Therefore, repetition and reinforcement of prerequisite material are essential to improve students' understanding *and minimize errors in solving contextual problems.*

INTRODUCTION

In high school, mathematics is one of the subjects that is often considered difficult by many students, especially in solving contextual problems(Putut, Emanuel, Maulina, & Soewardini, 2024). These difficulties do not merely stem from procedural complexity, but also from students' limited ability to connect mathematical concepts with real-life situations, interpret contextual information, and select appropriate solution strategies. As a result, students often rely on memorization rather than conceptual understanding when solving contextual problems. Mathematics learning in secondary schools related to contextual questions has an impact on learning in university mathematics (Lee, Ang, & Dipolog-Ubanan, 2019; Meehan & Howells, 2019). One of the materials that often causes difficulties is matrix inverses (Putut, Emanuel, & Anam, 2022). Matrix inverses have widespread applications in various fields, such as engineering, physics, and computer science. Therefore, a thorough understanding of this concept is crucial from an early age. However, many students struggle to solve problems involving matrix inverses, especially in real-world contexts that require a thorough understanding and application of mathematical principles.

At the university level, students must think creatively and critically in solving mathematical problems, and be able to communicate and elaborate on the material (Putut, Emanuel, Nusantara, & Rahardi, 2024). Students are expected to be able to understand mathematical ideas based on good communication skills(Fernández-León, Gavilán-Izquierdo, González-Regaña, Martín-Molina, & Toscano, 2019). Thinking is a form of communication between students and themselves and others (Sfard, 2008). Thinking and communicating are an inseparable unity (Presmeg, 2016). The combination of cognitive thinking and communication is known as commognitive. Commognitive consists of word use, visual mediators, narratives, and routines (Sfard, 2012). Words use (WU) can be words or terms spoken in everyday life, for example, the Y-axis, six, volume. WU can also be mathematical, for example, exponents, area, indefinite integrals. Visual mediators (VM) are visualization media that can be real objects, symbols, images, graphs, tables, or diagrams. Narratives (N) can be theorems, rules, for example, the Pythagorean theorem. Routines (R), which are repetitive patterns of steps in solving mathematical problems. These four components can be used in analyzing student work using a commognitive framework..

The commognitive framework has been widely used to analyze student work (Emanuel & Meilantifa, 2022). The purpose of this study is to describe students' work results when solving contextual problems on matrix inverses using the commognitive framework. The limitation of this study is to analyze incorrect work results. The results of this study are expected to make a significant contribution to the development of curriculum and teaching methods in mathematics education. By understanding students' thinking, teachers can adjust their teaching approaches to better support the learning process. This study can also serve as a basis for further research on the application of commognitive theory in the context of mathematics learning, as well as provide useful recommendations for the development of more effective teaching materials. By identifying patterns of commognitive conflict in students' incorrect work, this study addresses an important gap in existing research and provides insights into how prior knowledge, routines, and narratives influence students' problem-solving processes. The findings are expected to contribute to curriculum development and instructional design in mathematics education by enabling lecturers to design learning interventions that more effectively bridge secondary and tertiary mathematics learning.

THEORITICAL RIVIEW

The Commognitive Theory, developed by Anna Sfard, focuses on how communication and social interaction influence the process of learning mathematics. This theory states that mathematical thinking involves not only individual cognition but is also influenced by how students communicate and interact with mathematical contexts(Sfard, 2008). By understanding this process, we can better analyze the errors students make when solving contextual problems. In the context of mathematics education, communication plays a crucial role. Students need to be able to use mathematical language appropriately to explain their thinking. Misuse of mathematical terminology can lead to misunderstandings when solving contextual problems(Swidan & Daher, 2019). Therefore, analyzing how students communicate will provide insight into the difficulties they face.

Visual aids, such as graphs and diagrams, serve to help students understand complex mathematical concepts(Tabach & Nachlieli, 2016). In solving matrix inverse problems, the ability to use visual representations can be crucial to success. Narrative in the context of mathematics refers to the way students construct and explain mathematical arguments(Lu, Zhang, & Stephens, 2019). This includes the use of relevant definitions, theorems, and formulas. The ability to construct a coherent narrative can help students connect different concepts and solve problems more effectively.

A problem-solving routine includes the steps students follow in the problem-solving process(Thoma & Nardi, 2016). Certain patterns in these steps often lead to errors. By analyzing the routines for solving matrix inverse problems, common error patterns can be identified and strategies can be provided to help students avoid them. Conceptual errors often occur when students do not understand the basic principles underlying the material being studied(Kim et al., 2019). In the context of matrix inverses, this error can arise from a lack of understanding of the properties of matrices and how matrix inverses work. This error becomes apparent when analyzing student work.

The analysis of student work in solving problems using a commognitive framework is known as commognitive analysis. Commognitive analysis is conducted by considering four components: word use, visual mediators, narratives, and routines. **Table 1** shows the commognitive components and their descriptions.

Table 1. Commognitive Components

Commognitive Components	Sub Components	Descriptions	Examples
Words use (WU)	*Colloquial (WU-C)*	Using words and sentences in everyday life, both spoken and written.	A matrix is a collection of numbers arranged by rows and columns, and placed in brackets.
	Literate (WU-L)	Using words and sentences, symbols, signs, and numbers of a mathematical nature.	A matrix is an arrangement of numbers or functions arranged in rows (horizontal) and columns (vertical) to form a series of rectangles or squares, enclosed in brackets.
Visual Mediators (VM)	Concrete *(VM-C)*	Using real objects to solve problems	Using a rectangular object to calculate the area of a region
	Iconic (VM-I)	Using pictures or illustrations of objects, tables, and other visual representations to solve problems	Using graphs when illustrating problems
	Symbolic (VM-S)	Using symbols or variables to solve problems	$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$
Narratives	*Substantiation (NS)*	Using justification and reasoning to solve problems	$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$ has an inverse because the determinant of A is not zero
	Memorisation (NM)	Using formulas or rules to solve problems	$A.X = B \rightarrow X = A^{-1}.B$
Routines	*Ritualised (RR)*	Using previously implemented steps or patterns	The first step, determine the determinant of the matrix...if it is not zero, then the inverse exists. Then…
	Exploratory (RE)	Using new ways to	Determine the inverse of a

		solve problems	matrix using elementary row operations

METHOD

This research is a qualitative research with the main instrument being the researcher himself(Cresswel, 2013). The researcher designed, prepared, implemented, observed, analyzed, and drew conclusions. Other instruments included a written test sheet and an interview sheet. This research was conducted in the Mathematics Education study program, Wijaya Kusuma University, Surabaya. A total of 20 students participated in this study by working on a contextual problem about matrix inverses for 30 minutes individually. The problem given to the participants was as follows.

> Pada hari Minggu, Budi membeli 2 kg mangga, 1 kg jambu, dan 1 kg jeruk seharga Rp 50.000,00. Keesokan harinya, Budi membeli 1 kg mangga, 2 kg jambu, dan 1 kg jeruk dengan nominal pembayaran yang sama sehari sebelumnya. Ketika di hari Selasa, Budi membeli 3 kg mangga, 1 kg jambu, dan 1 kg jeruk, ia diharuskan membayar uang sebesar Rp 60.000,00. Ketika Ibu menanyakan harga masing-masing jenis buah per kilogram, tolong Anda bantu Budi menyelesaikan persoalan tersebut!
> Translate:
> On Sunday, Budi bought 2 kg of mangoes, 1 kg of guavas, and 1 kg of oranges for Rp 50,000.00. The next day, Budi bought 1 kg of mangoes, 2 kg of guavas, and 1 kg of oranges with the same payment amount as the day before. When on Tuesday, Budi bought 3 kg of mangoes, 1 kg of guavas, and 1 kg of oranges, he was required to pay Rp 60,000.00. When Mother asked the price of each type of fruit per kilogram, please help Budi solve the problem!

Student work was grouped based on whether their answers were correct. Incorrect work was then analyzed using a commognitive framework to identify patterns of errors. Two students were purposively selected as research subjects based on the characteristics of their incorrect work, which represented different types of commognitive difficulties, particularly inappropriate routines and conflicting narratives. These subjects were chosen because their solutions clearly illustrated distinct error patterns relevant to the research focus.

Each subject participated in a semi-structured interview to clarify their reasoning and decision-making processes during problem solving. The semi-structured interview format was chosen to allow flexibility in probing students' thinking while ensuring that they did not feel constrained or directly examined. The data collected included student work, interview transcripts, photographs, audio recordings, and videos. Data analysis was conducted through commognitive analysis and triangulation of written work, interview responses, and observational data, from which conclusions were drawn.

RESULTS AND DISCUSSION

The correct solution to the problem in this study is shown in Figure 1 as follows..

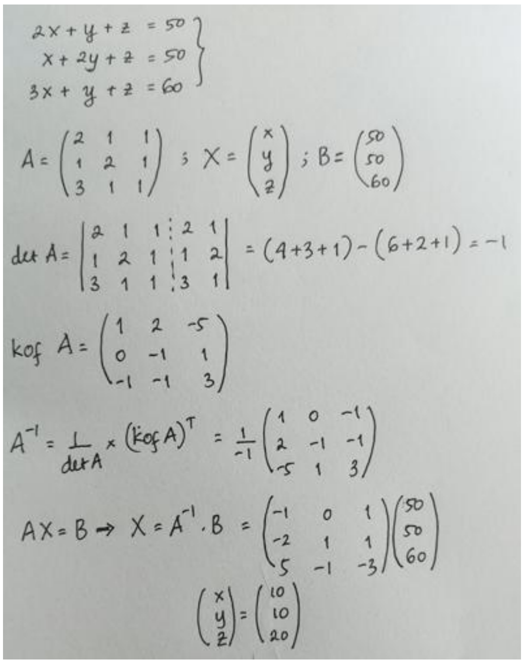

Figure 1 Correct answer to the research question

Subject S1 represents the first group who experienced errors in their work results, as shown in **Figure 2** below.

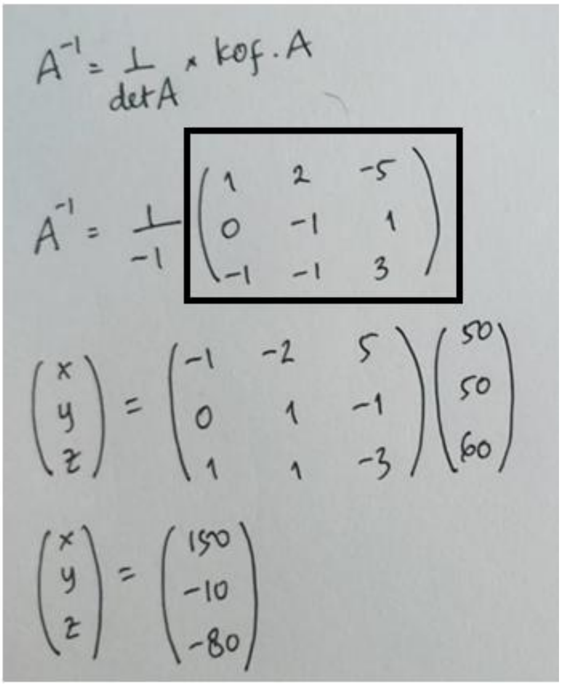

Figure 2 S1's work results

S1 was able to understand the questions well, including the correct answers. S1 was able to write matrix symbols correctly and write variables correctly. S1 was able to write the matrix determinant symbol and visualize the matrix determinant correctly. Based on his previous knowledge, S1 was able to determine the matrix determinant value correctly. When determining the cofactor matrix, S1 also did not make any errors because he understood the material on cofactors well. However, when determining the inverse matrix, S1 directly used the cofactor matrix without first inverting the cofactor matrix. This resulted in the obtained inverse matrix result being different from the correct answer.

When determining the solution to the problem, S1 found that two variables had negative values, and this confused him and made him doubt the accuracy of his answer. Clarification of S1's answer is shown in the following interview excerpt.

Q: Please explain your work! (pointing to S1's work)
S1: Okay, sir. Based on the given problem, I wrote a mathematical equation (WU). I wrote three equations with three variables (VM). Next, I wrote them in a 3 x 3 matrix (VM). Then, I calculated the determinant of the coefficient matrix, call it A, and the result was nonzero (NM).
P: Okay. What next?
S1: Next, I determined the cofactor matrix of A. Then, I determined the inverse matrix of A.
P: You determined the inverse matrix of A. How did you do that?
S1: I used the matrix inverse formula I learned in high school (RR-NM).
P: After that, what did you do?
S1: Next, I found the solution to the matrix equation, but I'm confused, why did the result come out negative? I doubt the accuracy of my answer. Is it possible for the price of an item to be negative?
P: When determining the inverse matrix, did you use the transpose of the cofactor matrix?

S1: Uhm, no sir. Oh, I remember now. The cofactor matrix should be transposed, sir. That's why my answer was negative. I understand now why my answer was wrong. Thank you very much, sir.
P: You're welcome.

Subject S2 represents the second group who experienced errors in their work results, as shown in **Figure 3** below.

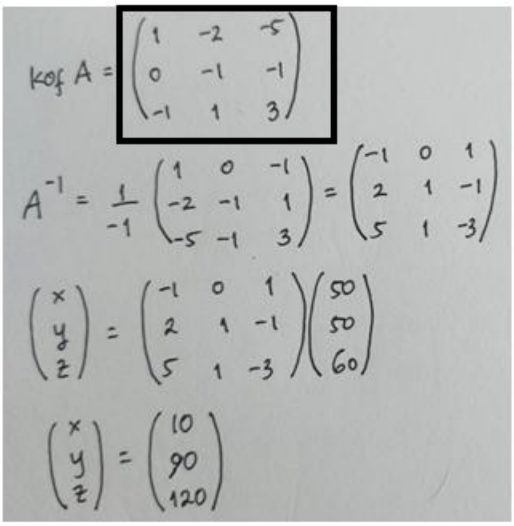

Figure 3 S2's work results

S2 was able to understand the questions well, including the correct answers. He was able to write the matrix symbols correctly and write the variables correctly. He was able to write the matrix determinant symbols and visualize the matrix determinant correctly. Based on his previous knowledge, he was able to determine the matrix determinant value correctly. When S2 determined the cofactor matrix, he made an error due to a lack of understanding of the material about cofactors. The error was made by S2 when determining the cofactor matrix, namely in elements whose row and column indices were odd numbers, not multiplied by negative one. This resulted in the obtained matrix inverse results being different from the correct answer.

When determining the solution to the problem, S2 was able to do it correctly. However, when re-examining the values of the three variables into the equation, the results were different, leaving S2 confused and doubting the correctness of his answer. Clarification of S2's answer is shown in the following interview excerpt.

Q: Please explain your work! (pointing to S2's work)
S1: Okay, sir. Based on the given problem, I wrote a mathematical equation (WU). I wrote three equations with three variables (VM). Next, I wrote them in a 3 x 3 matrix (VM). Then, I calculated the determinant of the coefficient matrix, called A, and the result was nonzero (NM).
Q: So, what was the next step?
S1: Next, I determined the cofactor matrix of A. Then, I determined the inverse matrix of A.
Q: You determined the cofactors of matrix A. How did you do that?

S1: I used a method for determining cofactors based on what I remember from high school material (RR-NM).
Q: After that, what did you do?
S1: Next, I looked for the solution to the matrix equation, but when I substituted it back into the problem, why did the results differ? I'm doubting the correctness of my answer, because the results weren't the same.
Q: When determining the cofactor matrix, did you pay attention to the number of row and column indices for each element? What about odd numbers? Even numbers?
S1: Uhm, no, sir. Oh, I remember now. Odd numbers should be multiplied by negative one, sir. That's why my answer didn't match when I checked it. I understand now why my answer was wrong. Thank you very much, sir.
P: You're welcome.

A good understanding of high school mathematics is essential when solving university math problems. Correct use of mathematical words, terms, and symbols (WU) can prevent errors. This opinion aligns with Ioannou(2018) which states that the correct use of words and terms can reduce errors in solving problems. In addition to the use of words (WU), the visual mediators used also play an important role in solving problems(Sharma, 2019). The use of symbols, variables, and mathematical notation (VM) also influences the steps in solving problems, namely making them easier to solve. This is in line with the opinion of Mpofu & Pournara(2018) which states that proper visualization can make it easier to solve mathematical problems.

Correctly sequenced steps or repetitive patterns taken when solving a problem also influence the correct solution (RR). Steps taken when solving a problem based on prior knowledge acquired in high school, if not mastered properly, can lead to incorrect answers(Çelik Demirci & Baki, 2023). Arguments presented during interviews indicated that prior knowledge due to a lack of understanding resulted in errors when solving problems (NM). Relying on prior knowledge but not mastering it well can result in incorrect answers. Errors in solving problems at university can be prevented by reviewing. Recalling material learned in high school can be done using student worksheets to reduce errors in solving problems.

CONCLUSION

Solving contextual mathematics problems at the university level requires the effective use of prior knowledge acquired in high school. However, insufficient understanding of prerequisite concepts often leads to errors in solving advanced mathematical problems. From a commognitive perspective, inaccuracies in word use, symbol selection, visualization, routines, and underlying narratives significantly influence students' problem-solving outcomes. Visualization plays a crucial role in guiding solution strategies, while well-structured routines and justified narratives are essential for ensuring correct reasoning. The findings of this study indicate that students' errors primarily stem from weak mastery of prior knowledge, which disrupts their commognitive processes and leads to inappropriate routines and conflicting narratives.

Therefore, it is strongly recommended that mathematics instruction at the university level explicitly reinforces prerequisite concepts through systematic repetition and targeted review of high school material. Lecturers should design learning activities that revisit key concepts related to matrix inverses, emphasize correct mathematical language and representations, and encourage students to reflect on their reasoning processes. Such instructional practices can help students recognize and resolve commognitive conflicts manifested as doubt or uncertaintythereby improving conceptual understanding and reducing errors in solving contextual mathematics problems.

DAFTAR PUSTAKA

Çelik Demirci, S., & Baki, A. (2023). Characterizing mathematical discourse according to teacher and student interactions: The core of mathematical discourse. *Journal of Pedagogical Research*, *7*(4), 144–164. https://doi.org/10.33902/JPR.202321852

Cresswel, J. (2013). Qualitative, quantitative, and mixed methods approaches. In *Research design*. https://doi.org/10.2307/3152153

Emanuel, E. P. L., & Meilantifa. (2022). Dimanakah Nilai Ekstrim Fungsi Kuadrat Ditinjau dari Lensa Commognitive ? *BRILIANT Jurnal Riset Dan Konseptual*, *7*(54), 269–279.

Fernández-León, A., Gavilán-Izquierdo, J. M., González-Regaña, A. J., Martín-Molina, V., & Toscano, R. (2019). Identifying routines in the discourse of undergraduate students when defining. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-019-00301-1

Ioannou, M. (2018). Commognitive analysis of undergraduate mathematics students' first encounter with the subgroup test. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-017-0222-6

Kim, D.-J., Choi, S., Lim, W., Thoma, A., Nardi, E., Viirman, O., … Sfard, A. (2019). Discourses of Functions – University Mathematics Teaching Through a Commognitive Lens. *Educational Studies in Mathematics*, *8*(2), 423–430. https://doi.org/10.1007/s10649-015-9676-1

Lee, K.-F., Ang, C.-S., & Dipolog-Ubanan, G. (2019). "My First Year in The University": Students' Expectations, Perceptions and Experiences. *Journal of Social Science Research*, *14*, 3134–3145. https://doi.org/10.24297/jssr.v14i0.8006

Lu, J., Zhang, X., & Stephens, M. (2019). Visualizing the commognitive processes in computer-supported one-to-one tutoring. *Interactive Learning Environments*. https://doi.org/10.1080/10494820.2019.1610452

Meehan, C., & Howells, K. (2019). In search of the feeling of 'belonging' in higher education: undergraduate students transition into higher education. *Journal of Further and Higher Education*, *43*(10), 1376–1390. https://doi.org/10.1080/0309877X.2018.1490702

Mpofu, S., & Pournara, C. (2018). Learner Participation in the Functions Discourse: A Focus on Asymptotes of the Hyperbola. *African Journal of Research in Mathematics, Science and Technology Education*, *22*(1), 2–13. https://doi.org/10.1080/18117295.2017.1409170

Presmeg, N. (2016). Commognition as a lens for research. *Educational Studies in Mathematics*, *91*(3), 423–430. https://doi.org/10.1007/s10649-015-9676-1

Putut, E., Emanuel, L., & Anam, F. (2022). Sebuah Tinjauan Commognitive : Apakah Matriks Singular ? *BRILIANT Jurnal Riset Dan Konseptual*, *7*(54), 922–930. https://doi.org/http://dx.doi.org/10.28926/briliant.v7i4.1073

Putut, E., Emanuel, L., Maulina, H., & Soewardini, D. (2024). Profil Komunikasi Matematis Tulis Mahasiswa Tahun Pertama Perkuliahan Dalam Pemecahan Masalah Differensial. *Briliant: Jurnal Riset Dan Konseptual*, *9*(3), 552–559.

Putut, E., Emanuel, L., Nusantara, T., & Rahardi, R. (2024). Does Domain of Absolute Value Function Always Positive ? Commognitive-Conflict Analysis of Students of First Semester. *Qalamuna: Jurnal Pendidikan, Agama, Dan Sosial*, *16*(2), 795–806. https://doi.org/10.37680/qalamuna.v16i2.5169

Sfard, A. (2008). Thinking as Communicating. In *Thinking as Communicating*. https://doi.org/10.1017/cbo9780511499944

Sfard, A. (2012). Introduction: Developing mathematical discourse-Some insights from communicational research. *International Journal of Educational Research*, *51–52*(December 2012), 1–9. https://doi.org/10.1016/j.ijer.2011.12.013

Sharma, S. (2019). Use of theories and models in geometry education research: A critical review. *Waikato Journal of Education*. https://doi.org/10.15663/wje.v24i1.644

Swidan, O., & Daher, W. M. (2019). Low achieving students' realization of the notion of mathematical equality with an interactive technological artifacts. *Eurasia Journal of Mathematics, Science and Technology Education*. https://doi.org/10.29333/ejmste/103073

Tabach, M., & Nachlieli, T. (2016). Communicational perspectives on learning and teaching mathematics: prologue. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-015-9638-7

Thoma, A., & Nardi, E. (2016). Routines in the didactical and mathematical discourses of closed-book examination tasks A commognitive analysis of closed-book examination tasks and lecturers' perspectives. *First Conference of International Network for Didactic Research in University Mathematics*.